

On the elliptical flow and asymmetry of the colliding nuclei

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Abstract

A study of elliptical flow is presented with respect to the asymmetry of colliding nuclei using the reactions of $^{24}Cr^{50} + _{44}Ru^{102}$, $^{16}S^{32} + _{50}Sn^{120}$ and $^{8}O^{16} + _{54}Xe^{136}$ at incident energies between 50 and 250 MeV/nucleon within the framework of isospin-dependent quantum molecular dynamics model. For the present analysis, total mass of the colliding pairs is kept fixed and asymmetry is varied as $\eta = 0.2, 0.3, 0.7$. The elliptical flow shows a transition from in-plane to out-of-plane in the mid rapidity region with incident energy. The transition energy is found to increase with the asymmetry for lighter fragments.

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I. INTRODUCTION

Large efforts are going on to understand the nuclear matter at the extreme conditions of temperature and density and also to explore the role of symmetry energy under these extreme conditions [1]. One of the most sought after phenomena in this direction is the collective flow and its various forms [2–5]. For the last few years, collective flow has been used as a powerful useful tool to explore the nuclear equation of state (EOS) as well as in-medium nucleon-nucleon cross-sections [1, 6]. Collective flow is a motion characterized by the space-momentum correlations of dynamic origin. Following two different signatures of collective flow have been predicted: a) bounce-off of compressed matter in the reaction plane [7] and b) squeeze-out of the participant matter out of the reaction plane [8].

Such observables together represent the anisotropic part of the transverse flow that appears in the non-central heavy-ion collisions only. The highly stopped and compressed nuclear matter around the mid-rapidity region is seen directly in the squeeze out [9], also known as elliptical flow. The elliptical flow has been proven to be one of the most fruitful probes to study the dynamics of heavy-ion collisions. The elliptical flow describes the eccentricity of an ellipse-like distribution. Quantitatively, it is the difference between the minor and major axis. The orientation of the major axis is confined to the azimuthal angle ϕ or $\phi + \frac{\pi}{2}$ for an ellipse-like distribution. The major axis lies within the reaction plane for ϕ ; whereas $\phi + \frac{\pi}{2}$ indicates that the orientation of the ellipse is perpendicular to the reaction plane (i.e., squeeze-out flow). The parameter of the elliptical flow is quantified by the second order Fourier coefficient $\langle v_2 \rangle = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle$, p_x and p_y being the x and y components of the momentum respectively), from the azimuthal distribution of detected particles at mid-rapidity [10]:

$$\frac{dN}{d\phi} = p_0(1 + 2v_1 \text{Cos}\phi + 2v_2 \text{Cos}2\phi).$$

where ϕ is the azimuthal angle between the transverse momenta of the particles and reaction plane. The positive values of elliptical flow reflect an in-plane emission, whereas out-of-plane emission is reflected by the negative values. The parameters $\langle \text{Cos}2\phi \rangle$ of elliptical flow depend on the complex interplay between the expansion, rotation and shadowing of the spectators, apart from the incident energy. It is worth mentioning that both the mean field

and binary nucleon-nucleon collision parts play an important role at intermediate energies. The mean field potential plays a dominant role at low incident energies, which is gradually taken over by the two body collision part at higher incident energies. Therefore, a detailed study on the excitation function of elliptical flow in this energy range can provide useful information about the nucleon-nucleon interactions and origin of the isospin effects in heavy-ion collisions.

It is worth mentioning that the outcome of a reaction depends also on the asymmetry of the reaction. Unfortunately, very few investigations focus on this aspect. The asymmetry of a reaction can be defined by the asymmetry parameter $\eta = (A_T - A_P)/(A_T + A_P)$; [11] where A_T and A_P are the masses of the target and projectile, respectively. The $\eta = 0$ corresponds to the symmetric reactions, whereas non-zero values of η define different asymmetries of a reaction. As noted by FOPI group [12–17], the reaction dynamics in a symmetric reaction ($\eta = 0$) can be quite different compared to an asymmetric reaction ($\eta \neq 0$). This is valid both at low and intermediate energies. This difference emerges due to the different deposition of the excitation energy (in form of compressional and thermal energies) in symmetric and asymmetric reactions. Though the systematic role of asymmetry has been explored in multifragmentation, no such study yet exists in the literature for elliptical flow [18]. We plan to address this in present paper.

We plan to understand how the elliptical flow is affected by the asymmetry of a reaction. This study is performed within the isospin-dependent quantum molecular dynamics (IQMD) model discussed in section II. Our results are presented in section III. Finally, we summarize the results in section IV.

II. THE MODEL

The isospin-dependent quantum molecular dynamics (IQMD)[17–19] model treats different charge states of nucleons, deltas and pions explicitly, as inherited from the Vlasov-Uehling- Uhlenbeck (VUU) model. The IQMD model has been used successfully for the analysis of a large number of observables from low to relativistic energies. The isospin degree of freedom enters into the calculations via symmetry potential, cross sections and Coulomb interactions.

In this model, baryons are represented by Gaussian-shaped density distributions

$$f_i(r, p, t) = \frac{1}{\pi^2 \hbar^2} e^{\frac{-(r-r_i(t))^2}{2L}} e^{\frac{-(p-p_i(t))^2 \cdot 2L}{\hbar^2}}. \quad (1)$$

where L is the Gaussian width which is taken to be 2.16 fm^2 . In ref [19], this Gaussian width is found to be dependent on the size of the system. Nucleons are initialized in a sphere with radius $R = 1.12A^{1/3} \text{ fm}$, in accordance with the liquid drop model. Each nucleon occupies a volume of \hbar^3 so that phase space is uniformly filled. The initial momenta are randomly chosen between 0 and Fermi momentum p_F . The nucleons of the target and projectile interact via two and three-body Skyrme forces, Yukawa potential and Coulomb interactions. The isospin degrees of freedom is treated explicitly by employing a symmetry potential and explicit Coulomb forces between protons of the colliding target and projectile. This helps in achieving the correct distribution of protons and neutrons within the nucleus. The hadrons propagate using Hamilton equations of motion:

$$\frac{d\vec{r}_i}{dt} = \frac{d \langle H \rangle}{dp_i} \quad ; \quad \frac{d\vec{p}_i}{dt} = -\frac{d \langle H \rangle}{dr_i}. \quad (2)$$

with

$\langle H \rangle = \langle T \rangle + \langle V \rangle$ is the Hamiltonian, which is written as:

$$= \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\vec{r}, \vec{p}, t) V^{ij}(\vec{r}', \vec{r}) f_j(\vec{r}', \vec{p}', t) d\vec{r} d\vec{r}' d\vec{p} d\vec{p}'. \quad (3)$$

The baryon-baryon potential V^{ij} , in the above relation, reads as

$$\begin{aligned} V^{ij}(\vec{r}' - \vec{r}) &= V_{Skyrme}^{ij} + V_{Yukawa}^{ij} + V_{Coul}^{ij} + V_{Sym}^{ij} \\ &= t_1 \delta(\vec{r}' - \vec{r}) + t_2 \delta(\vec{r}' - \vec{r}) \rho^{\gamma-1} \left(\frac{\vec{r}' + \vec{r}}{2} \right) \\ &+ t_3 \frac{\exp(|\vec{r}' - \vec{r}|/\mu)}{(|\vec{r}' - \vec{r}|/\mu)} + \frac{Z_i Z_j e^2}{|\vec{r}' - \vec{r}|} \\ &+ t_4 \frac{1}{\rho_o} T_3^i T_3^j \cdot \delta(\vec{r}' - \vec{r}_j). \end{aligned} \quad (4)$$

Where $\mu = 0.4 \text{ fm}$, $t_3 = -6.66 \text{ MeV}$ and $t_4 = 100 \text{ MeV}$. The values of t_1 and t_2 depends on the values of α , β , and γ [1]. Here Z_i and Z_j denote the charges of the i^{th} and j^{th} baryon, and T_3^i , T_3^j are their respective T_3 components (i.e. $1/2$ for protons and $-1/2$ for

neutrons). The Meson potential consists of Coulomb interaction only. The parameters μ and t_1, \dots, t_4 are adjusted to the real part of the nucleonic optical potential.

III. RESULTS AND DISCUSSIONS

For a controlled study of the role of asymmetry of a reaction, the total reacting mass is fixed equal to 152 units. While total mass stayed constant, asymmetry η is varied by choosing different combinations of projectile-target. We shall perform exclusive studies by simulating the reactions of $^{24}Cr^{50} + _{44}Ru^{102}$ ($\eta = 0.3$), $^{16}S^{32} + _{50}Sn^{120}$ ($\eta = 0.5$), and $^{8}O^{16} + _{54}Xe^{136}$ ($\eta = 0.7$) at incident energies between 50 and 250 MeV/nucleon for semi-central impact parameter using a soft equation of state. The phase space generated by the IQMD model is analyzed using the minimum spanning tree (MST) [1] method. This method binds two nucleons in a fragment if their distance is less than 4 fm.

As stated above, positive value of elliptical flow describes the eccentricity of an ellipse-like distribution and indicates in-plane enhancement of the particle emission, i.e., rotational behavior. On the other hand, a negative value of $\langle v_2 \rangle$ shows the squeeze-out effects perpendicular to the reaction plane. Obviously, zero value corresponds to an isotropic distribution. Generally, for a meaningful understanding v_2 is extracted from the midrapidity region only. Naturally, midrapidity region corresponds to the collision (participant) zone and hence signifies compressed matter. On the other hand, $Y_{c.m.}/Y_{beam} \neq 0$ corresponds to spectator region; $Y_{c.m.}/Y_{beam} < -0.1$ corresponds to target like (TL) matter whereas $Y_{c.m.}/Y_{beam} > 0.1$ corresponds to projectile like (PL) matter.

In Fig.1, final state elliptical flow is displayed for free particles (upper panel) and light charged particles (LCP's) [$(2 \leq A \leq 4)$] (lower panel) as a function of transverse momentum (P_t). One can see a Gaussian shaped behavior at all asymmetries quite similar to the one reported by Colona and Toro *et al.*, [20]. Note that the Gaussian shaped behavior is integrated over the entire rapidity range. One also sees that elliptical flow of nucleons/LCP's is positive over entire P_t range. One also notices that for larger asymmetries (e.g. $\eta = 0.7$), prominent peaks of Gaussian noted earlier diminishes. There is a linear increase in the elliptical flow with transverse momentum upto certain incident energy. Obviously, particles with larger momentum will escape the reaction zone at an earlier time. After certain value,

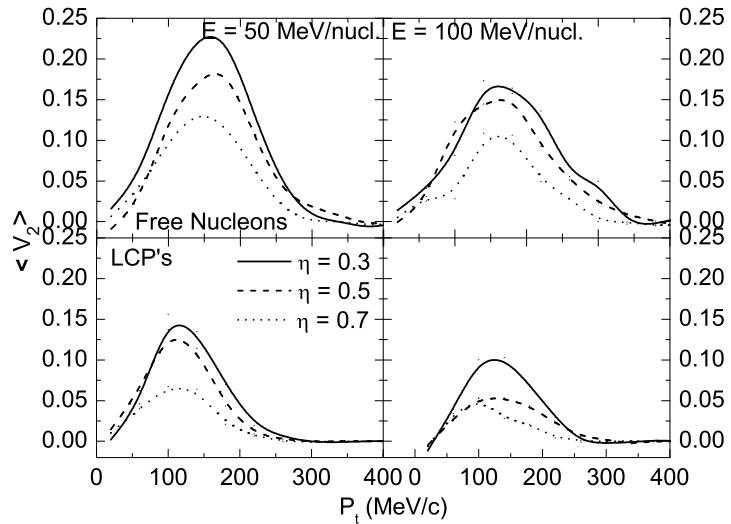


FIG. 1: The transverse momentum dependence of elliptical flow, summed over all rapidity bins at $\hat{b} = 0.3$ for different asymmetries at 50 (left) and 100 (right) MeV/nucleon. The upper and lower panels represent the free nucleons and light charged particles (LCP's), respectively.

elliptical flow decreases indicating that not enough particles occupy such higher P_t values. One also notices lower peaks of Gaussian for heavier fragments. The reason for this shift is that the emitted free nucleons can feel the role of mean-field directly, while the LCP's have a weaker sensitivity [21]. Interestingly, $\langle v_2 \rangle$ depends significantly on the symmetry of the reaction also. Further, we see that neutron-rich system (${}^8O^{16} + {}^{54}Xe^{136}$ with $N/Z = 1.4$) exhibits a weaker squeeze-out flow compared to other reactions. These findings are in agreement with the one reported by Zhang *et al.*, [22]. Moreover, the N/Z effect is more pronounced at $E = 50$ MeV/nucleon. This also indicates that in addition to the mass asymmetry, the mean field effects such as isospin effects are also responsible for different elliptical flow.

In Fig. 2, we divide the total elliptical flow into contributions from target-like (TL), mid-rapidity, and projectile-like (PL) particles. From the figure, we see that the projectile-like (PL) nucleons and LCP's feel more squeeze out compared to target-like (TL) nucleons/LCP's. Due to larger asymmetry, only small fraction of nucleons/LCP's experience squeeze out compared to symmetric reactions. This decrease of squeeze out with asymmetry happens due to decreasing participant zone. This is in agreement with earlier calculations

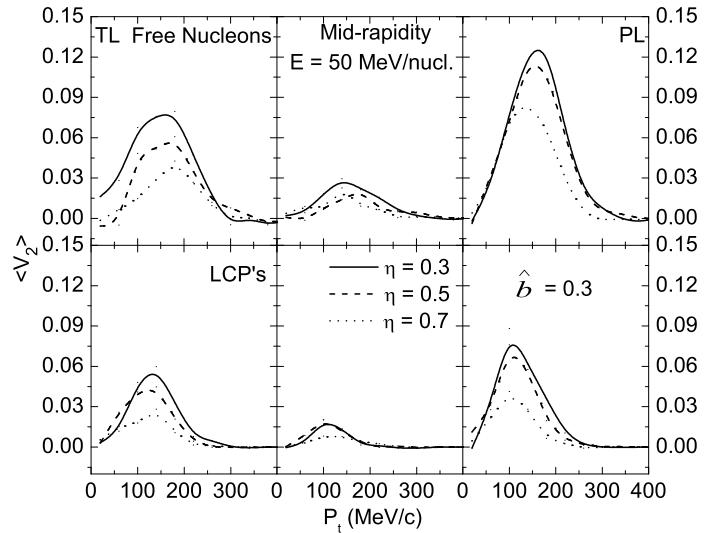


FIG. 2: The transverse momentum dependence of the elliptical flow at $E = 50$ MeV/nucleon for different asymmetries divided into contributions from target-like, midrapidity and projectile-like matter, respectively; the upper and lower panels have same meaning as in Fig. 1.

where fragments were found to exhibit similar trends.

Since the change of the asymmetry of a reaction can affect the elliptical flow via symmetry energy, we display in Fig. 3, the transverse momentum dependence of elliptical flow for LCP's in the mid-rapidity region for the reaction of $^{24}Cr^{50} + _{44}Ru^{102}$ with and without symmetry energy. The effect of symmetry energy is clearly visible. This is in agreement with the findings of Chen *et al.*, [23], where it was concluded that the production of LCP's act as a probe for symmetry energy.

In Fig. 4, we display the variation of excitation function $\langle v_2 \rangle$ for LCP's as a function of incident energy for entire rapidity region and for mid rapidity region $-0.1 \leq Y^{red} \leq 0.1$ only. The general behavior of excitation functions for various asymmetries is quite similar. The microscopic behavior, however, depends on the asymmetry of the reaction. Interestingly, no transition in the elliptical flow occurs when entire rapidity region is considered. In contrast, a transition from the preferential in-plane (rotational like) emission ($v_2 > 0$) to out-of-plane (squeeze-out) emission ($v_2 < 0$) occurs at mid rapidity zone. This happens due to the fact that the contribution of spectator matter increases with rapidity region,

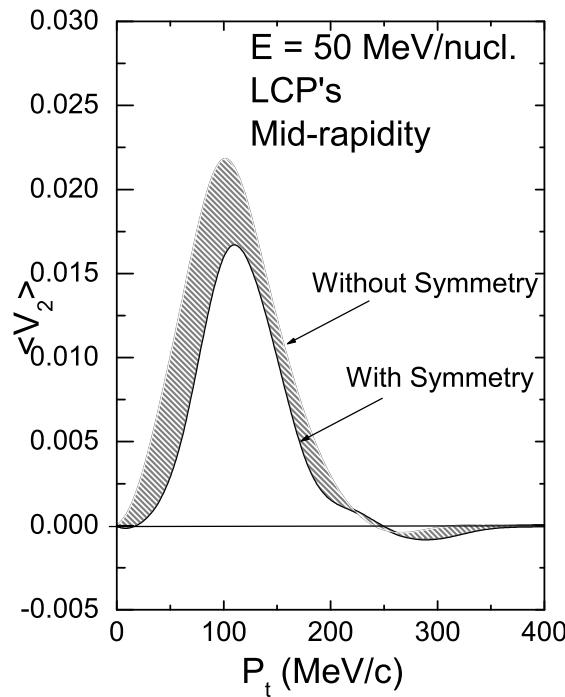


FIG. 3: The transverse momentum dependence of elliptical flow for LCP's in the midrapidity region at $E = 50$ MeV/nucleon. The panel exhibits the effect of symmetry energy on $^{24}Cr^{50} + _{44}Ru^{102}$ reaction at $\hat{b} = 0.3$.

leading to less squeeze-out of the particles in entire rapidity region. On the other hand, the contribution of the participant zone dominates the reaction in midrapidity region leading to the transition from in-plane to out-of-plane. This happens because the mean field which contributes to the formation of a rotating compound system becomes less important and collective expansion process (based on the nucleon-nucleon binary scattering) starts to be predominant [24]. The competition between the mean-field and nucleon-nucleon collisions should strongly depend on the effective interactions, which leads to the divergence of the transition energies calculated by varying the asymmetry of a reaction. In other words, participant zone is primarily responsible for the transition from in-plane to out-of-plane. The energy at which this transition is observed is dubbed as the transition energy E_{trans} . That is why, LCP's, which originate from the participant zone, show a clear and systematic transition with the beam energy as well as with the asymmetry of a reaction. One should note that transition energies increases with the asymmetry of a reaction.

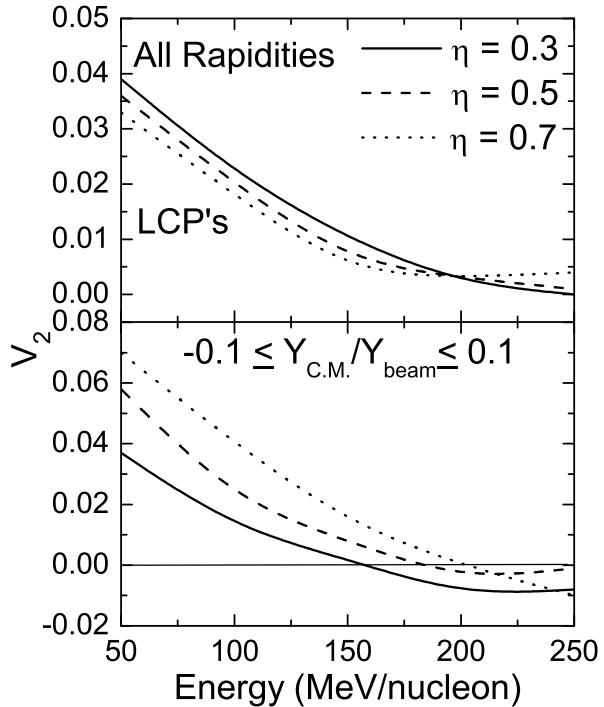


FIG. 4: The variation of elliptical flow (summed over the entire transverse momentum) with beam energy at $\hat{b} = 0.3$ for different asymmetries over the entire rapidity range (left panels) and at midrapidity (right panels). The upper and lower panels have same meaning as in Fig. 1.

IV. CONCLUSION

In the present study, elliptical flow is studied for different asymmetries leading to same compound masses. For this, the reactions of $^{24}Cr^{50} + _{44}Ru^{102}$ ($\eta = 0.3$), $^{16}S^{32} + _{50}Sn^{120}$ ($\eta = 0.5$), and $^{8}O^{16} + _{54}Xe^{136}$ ($\eta = 0.7$) are simulated at incident energies between 50 and 250 MeV/nucleon using isospin-dependent quantum molecular dynamics model. The characteristic features of the elliptical flow are described by varying the mass asymmetry. The elliptical flow is found to show a transition from in-plane to out-of-plane in the mid rapidity region with incident energy. The transition energy at which the elliptical flow v_2 changes sign from positive to negative are different for different asymmetries. The transition energy is found to increase with the asymmetry for lighter fragments.

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